6.4b Exponential Growth and Decay

Newton's Law of Cooling: The rate at which an object's temperature is changing is directly proportional to the difference between its temperature and the temperature of the surrounding medium.

$\frac{dT}{dt}$

T = Temp T_s = Temp of Surrounding t = time

$\frac{dT}{dt} = -k(T-T_s)$

$\int \frac{dT}{T-T_s} = \int -k\,dt$

$\ln|T-T_s| = -kt + c$

$e^{\ln|T-T_s|} = e^{-kt+c}$

$|T-T_s| = e^{-kt} \cdot e^{c} = e^{-kt} \cdot A$

$t=0$ $T=T_0$ beginning Temp

$T_0 - T_s = e^0 \cdot A = A$

$T-T_s = (T_0-T_s)e^{-kt}$

$T = (T_0-T_s)e^{-kt} + T_s$

$y = 105(.997)^x$

$y = ab^x$

Dec 6-10:18 PM

A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

$T = (T_0 - T_s)e^{-kt} + T_s$

$T = (98-18)e^{-kt} + 18$

$38 = 80e^{-k\cdot 5} + 18$

$20 = 80e^{-5k}$

$\frac{20}{80} = e^{-5k}$

$\ln\frac{1}{4} = -5k$

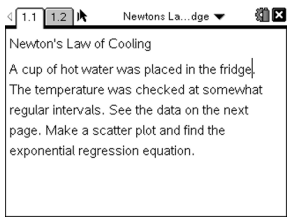
$k = \frac{\ln .25}{-5} \approx .277$

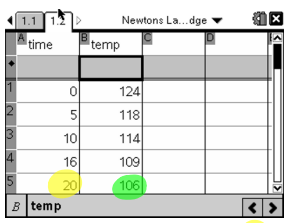
$20 = 80e^{-.277t} + 18$

$t = 13$

$13 - 5 = 8$ min longer

Dec 6-10:23 PM





$T_0 = 124$ $T_s = 45$

$T = (124-45)e^{-kt} + 45$

$106 = 79e^{-k\,20} + 45$

$k = .013$

$T = 79e^{-.013t} + 45 = 79(.987)^t + 45$

calc $T = 70.8(.990833)^x + 45$

Dec 6-10:25 PM

Separation of Variables

Solve for y if $\frac{dy}{dx} = (xy)^2$ and y=1 when x=1

RTC!!

$\frac{dy}{dx} = x^2y^2$

$\int \frac{dy}{y^2} = \int x^2\,dx$

$\int y^{-2}\,dy = \int x^2\,dx$

$-y^{-1} = \frac{x^3}{3} + c$

$-1 = \frac{1}{3} + c$ $c = -\frac{4}{3}$

$\frac{-1}{y} = \frac{x^3}{3} - \frac{4}{3} = \frac{x^3-4}{3}$

$\frac{1}{y} = \frac{-x^3+4}{3}$

$y = \frac{3}{4-x^3}$

Dec 6-10:49 PM